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Taming the supergravity description of non-BPS D-branes: the D/$\bar{\text{D}}$ solution.

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Abstract: We obtain the supergravity solution which describes a bound state of D-string/anti-D-string pairs attached to different fixed planes of an orbifold, in type-IIB string theory compactified on $T^4/\mathbb{Z}_2$. For parameters at which the conformal field theory point of view predicts stability, the solution displays a repulson-like singularity. However, we observe that a D-string/anti-D-string pair probe in this background becomes tensionless before reaching the singularity, suggesting a resolution by the enhançon mechanism. Moreover, the force feels by this probe is attractive, in contrast to the repulsive behaviour observed in the non-BPS D-brane description.

Keywords: Solitons Monopoles and Instantons, D-branes, Superstring Vacua.

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Contents

1. Introduction

Non-BPS D-branes have received a lot of attention in the last couple of years as a way to test duality symmetries between string theories beyond the supersymmetric spectrum [1]–[9]. These studies mainly rely on the conformal field theory description of the non-BPS branes, as dynamical surfaces on which open strings end [10]. However, we know that BPS-branes also enjoy a space-time description as classical solutions of the field equations of supergravity [11]. While the conformal field theory description is valid at weak coupling, the supergravity solution is a good description of a strong interacting system, especially for a large number of D-branes.

A crucial ingredient for superposing an arbitrary number of BPS D-branes is the so-called "no-force" condition, which is a consequence of the presence of preserved supersymmetries in this context: since these D-branes do not exercise any force on each other, one can bring N of them from infinity to form a macroscopic bound state which has a reliable classical description.

On the other hand, in general, stable non-BPS D-branes do not obey such a constraint. Actually, only non-BPS D-branes of orbifold compactification of type-II string theories have been found to enjoy this property at one-loop [14]. Indeed, for specific radii of compactification, a Bose-Fermi degeneracy appears and ensures that the no-force condition is satisfied at this order in the coupling constant. Therefore, assuming that such D-branes should have a classical description in supergravity, several papers tried to find the explicit solution [16]–[18] (see also [19] for a study in the ten dimensional context). They found that, as expected, the solution verifies the no-force condition at one-loop for a critical volume but, at higher orders, the potential felt by a non-BPS D-brane probe in the background geometry created by the other D-branes is not constant: the no-force condition fails and these non-BPS branes actually exercise a repulsion on each other at any distance, which prevents them to form a bound state [18]. Moreover, the metric is plagued with naked singularities, located at finite proper distances from the core of the brane, and no mechanism to hide them has been found so far.

More recently, Lambert and Sachs [20] argued that, at critical radii, the non-BPS D-brane states considered in [16, 18] were not the minimum of the tachyon potential. The true vacuum is obtained when the world-volume scalars associated to the tachyon winding modes condense; in the type-II string theory compactified on $T^4/\mathbb{Z}_2$ picture, it is described by a system of D-brane/anti-D-brane located at two different fixed points of the orbifold, as we will review in detail in the next section. They calculated the one-loop potential and they found that, in this phase, two non-BPS branes attract each other, independently of the K3 volume. Therefore, it seems to be a good starting point to find a classical description. The aim of the present article is to provide such a solution for the D-string/anti-D-string system in type-IIB theory on $T^4/\mathbb{Z}_2$. However, the calculation could be extended easily to other Dp-branes configurations, at least for p less than three. Indeed, for $p = 3$, we do not expect anymore an asymptotically flat space, since the supergravity fields will have a logarithmic dependence on the radial coordinate.

This paper is organized as follows. In the next section 2, we review the conformal field theory description of non-BPS D-branes in type-IIB string theory on $T^4/\mathbb{Z}_2$. In particular, we give their associated boundary states which will be useful to obtain the boundary action and the asymptotic behaviour of the supergravity solution. We also review the boundary state obtained at the minimum of the tachyon potential, as discussed in [20]. In section 3 three, we write the six-dimensional supergravity lagrangian which describes the dynamics of the fields excited by the D-brane configurations. We also discuss the boundary actions which reproduce the linear couplings of the supergravity fields to the non-BPS D-branes. Another useful information given in this section is the asymptotic behaviour of the bulk fields in the presence of a non-BPS D-brane or of a D-string/$\bar{\text{D}}$-string pair. The subject of this section has a significant overlap with [21], which appears while our work was in progress. Therefore, to avoid repetitions, we will borrow its notations and results and extend them to the non-BPS configurations.

In section 4, we first discuss the issue of the no-force condition in our configuration and then solve the equations of motion. Details are given in an appendix. We observe that the solution, which depends on the moduli of K3 only through its volume, always displays naked singularities. However, by studying the behaviour of a slowly moving probe in this background, we see that the brane becomes tensionless before reaching the first singularity. Therefore, we argue that an enhançon mechanism similar to the one advocated in [22] should provide a resolution of the singularities. In particular, we show that, on the tensionless sphere, new massless bulk states are present and the gauge symmetry is enhanced. Finally, the last section 5 contains some comments and discusses a related problem.

2. The non-BPS D2-brane and the D-string/anti-D-string system in type-IIB string theory on $T^4/\mathbb{Z}_2$

2.1 The supergravity

The context of our study is type-IIA string theory compactified on $T^4/(-1)^{F_L}\mathcal{I}_4$ where F_L is the left-mover spacetime fermion number. By T-duality along one of the orbifold directions, this theory is related to the type-IIB string theory on $T^4/\mathcal{I}_4$. In this paper, we will not try to find supergravity solutions in ten dimensions, with D-branes localized at some fixed points of the orbifold. Indeed, such study would involve the construction of dipoles of (fractional) D-branes, dipoles which are not yet well understood, even in the context of standard BPS D-branes in ten dimensions: so far, only dipoles solutions depending of three transverse directions have been discussed in the literature [23]–[26]. Moreover, the supergravity description of fractional D-branes has appeared only recently, first for D3-branes on a non-compact $\mathbb{Z}_2$ orbifold [27] and then, for D0-branes in type-IIA string theory compactified on $T^4/\mathbb{Z}_2$ [21].

The six-dimensional low energy effective theory is the chiral $\mathcal{N}_6 = (2,0)$ supergravity and contains the graviton multiplet and twenty-one tensor multiplets. From the type-IIB point-of-view, the bosonic content of these multiplets comes from the reduction of the ten-dimensional NS-NS and R-R fields on K3. The reduction of the metric gives the six-dimensional metric tensor g and 58 scalars. In the following, we will be particularly interested into the four scalars which correspond to the diagonal elements of the metric along the compact directions and which will be denoted η^a, as in [18, 21]. The reduction of the NS-NS 2-form on the supersymmetric 2-cycles of K3 gives 22 scalars, which can be decomposed into 6 untwisted states associated to the 2-cycles of the 4-torus and 16 twisted moduli, called b^I ($I = 1, \ldots, 16$) below and which are associated to the 16 exceptional supersymmetric 2-cycles of K3. In the orbifold limit, these twisted fields are localized at the 16 fixed points of the orbifold. From the R-R sector, one obtains R-R twisted 2-forms and twisted scalars. Explicitly, the ten dimensional R-R 4 and 2-form, $C_{(4)}$ and $C_{(2)}$, decompose as:

$$\begin{aligned} C_{(4)} &= C^6_{(4)} + \sum_{i=1}^{6} C^i_{(2)} \wedge \omega^i_2 + \sum_{i=1}^{16} \mathcal{A}^I_{(2)} \wedge \tilde{\omega}^I_2 + \tilde{C}_{(0)}\,\omega_4 \\ C_{(2)} &= C^6_{(2)} + \sum_{i=1}^{6} C^i_{(0)}\,\omega^i_2 + \sum_{i=1}^{16} \mathcal{A}^I_{(0)}\,\tilde{\omega}^I_2\,, \end{aligned} \tag{2.1}$$

where ω^i_2, $\tilde{\omega}^I_2$ and ω_4 denote respectively the harmonic forms dual to the six 2-cycles of the 4-torus, to the sixteen vanishing 2-cycles and to the 4-cycle. Notice that the 4-form $C^6_{(4)}$ can be Hodge-dualized to a scalar. Taking into account the anti-self-duality of the exceptional 2-cycles $\tilde{\omega}^I_2$, the ten dimensional self-duality of the field strength of $C_{(4)}$ translates into anti-self-duality conditions for the field strengths of the 2-forms $\mathcal{A}^I_{(2)}$. In the following, we will forget the "6" index since we will work exclusively in six dimensions.

Contrary to the non-chiral theory considered in [18], the six-dimensional lagrangian which describes the dynamics of this chiral $\mathcal{N}_6 = (2,0)$ supergravity can not be obtained from the Kaluza-Klein reduction of a string theory on a 4-torus. Therefore, we will adopt another method, also used in [21] in the context of type-IIA string theory on $T^4/\mathbb{Z}_2$. Moreover, we will not write the full lagrangian, but only the terms which are relevant for our study, i.e. the bulk fields excited by the D-string/anti-D-string sources. To select these fields, we first have to describe precisely the sources, what we will do in the next two subsections.

2.2 The non-BPS D-brane

We will limit our study to the non-BPS D-string of type-IIA compactified on the orbifold $T^4/(-1)^{F_L}\mathcal{I}_4$. This D-string, which is localized at one of the sixteen fixed points of the orbifold space, say $\mathbf{x}_1 = (0,0,0,0)$, is stable provided that all radii, $R_a, a = 6,\ldots,9$, of the four-torus are larger than $\sqrt{\alpha'/2}$.[1] Indeed, the tachyonic ground state of the NS sector of open strings is projected out by $(-1)^{F_L}\mathcal{I}_4$ and only odd winding modes survive. Therefore, the square masses of the four lightest states,

$$\frac{\chi^a}{2i}\left(e^{iX^aR_a/\alpha'} - e^{-iX^aR_a/\alpha'}\right), \qquad \alpha' m_a^2 = -\frac{1}{2} + \frac{R_a^2}{\alpha'}\,, \qquad a = 6,\ldots,9$$

are positive for $R_a \geq \sqrt{\alpha'/2}$.

When one of these radii, for instance R_9, becomes smaller that this critical value, the field χ^9 is tachyonic; the non-BPS D-string is unstable and decays into a D2/$\bar{\text{D}}$2-branes pair, stretched between the two fixed points along the x^9 direction, with a $\mathbb{Z}_2$

[1] Latin indices $a, b, \ldots$ will denote the four compact coordinates $x^6, \ldots, x^9$. We will also use greek indices $\alpha, \beta, \ldots$ for the non-compact longitudinal coordinates and latin letters $i, j, \ldots$ for the non-compact transverse coordinates.

Wilson line on one of the D2-branes [12, 13]. Moreover, when all radii are critical, the non-BPS D-string is stable and its spectrum has a Bose-Fermi degeneracy [14]. Therefore, the one-loop open string partition function vanishes and the no-force condition between two such non-BPS objects is verified at this order.

From the conformal field theory point-of-view, this non-BPS D-string is described by a boundary state made up of an untwisted NS-NS part and a twisted R-R part [13]:

$$|D1\rangle = \frac{T_1}{\sqrt{2V}} |B1, \mathbf{x}_1\rangle^U_{\mathrm{NS-NS}} + \frac{T_1}{\sqrt{2\pi^2\alpha'}} |T_1\rangle_{\mathrm{R-R}} \,. \tag{2.2}$$

The explicit form of the boundary states $|B1, \mathbf{x}_1\rangle^U_{\mathrm{NS-NS}}$ and $|T_1\rangle_{\mathrm{R-R}}$ can be found for instance in [21].

In the T-dual type-IIB language, this non-BPS state corresponds to a non-BPS D2-brane stretched between the two fixed planes along the T-dualized direction [9]. After T-duality, the new stability constraint on the radii reads $R_a \geq \sqrt{\alpha'/2}$ for $a = 6, 7, 8$ and $R_9 \leq \sqrt{2\alpha'}$, if x^9 is the T-dualized direction. When R_9 becomes larger than $\sqrt{2\alpha'}$, the non-BPS D2-brane decays into a fractional D-string/anti-D-string pair, each localized at opposite fixed points, $\mathbf{x}_1 = (0, 0, 0, 0)$ and $\mathbf{x}_2 = (0, 0, 0, \pi R_9)$ along the x^9 direction [5, 15]. The boundary state which corresponds to this non-BPS D2-brane is given by:

$$|\hat{D2}\rangle = \frac{T_1 R_9}{2\sqrt{V\alpha'}} |B2\rangle^U_{\mathrm{NS-NS}} + \frac{T_1}{2\sqrt{2\pi^2\alpha'}} \left(|T_1\rangle_{\mathrm{R-R}} + |T_2\rangle_{\mathrm{R-R}}\right) . \tag{2.3}$$

Other decay channels, obtained when $R_a < \sqrt{\alpha'/2}$, are described in [9].

Using these boundary states, one can first extract the linear couplings of the bulk fields to the D-branes, by evaluating one-point functions, and, second, find the asymptotic behaviour of these fields at large distance from the D-branes [28]. The method has been used in the context of the non-BPS D-particle of type-IIB string theory compactified on $T^4/(-1)^{F_L}\mathcal{I}_4$ in [16] to obtain the large distance dependence of the bulk fields. Since these calculations encode only the one-loop effect of string theory, one recovers the no-force condition when the radii are critical.

However, to find the complete solution and not only the asymptotic behaviour, it is easier to solve directly the equations of motion with the appropriated source terms rather than to calculate higher loops diagrams. This program has been settled in [18], where they considered a source made of a superposition of N non-BPS D0-branes in type IIB on $T^4/(-1)^{F_L}\mathcal{I}_4$ and they succeeded to find the complete solution which described this state. Their solution, which is exact in the large t'Hooft coupling ($Ng_s \to \infty$), exhibits pathological behaviours: first, their solution displays naked repulson-like singularities which have not been cured so far; the enhançon mechanism which has recently been put forward to solve similar singularities in other contexts does not seems to apply here since characteristic properties such as the existence of a tensionless locus or the enhancement of gauge symmetries are missing in their solution. Moreover the no-force condition is not satisfied for higher string loops than one-loop; actually, the force between the non-BPS branes is always repulsive, preventing them to form a localized bound state and, therefore, contradicting the initial hypothesis. This implies that one can not construct a reliable supergravity solution with small enough curvatures to neglect the α' corrections to the bulk supergravity. Although the result is disappointing from the point of view of the quest of a classical description for non-BPS D-branes, the supergravity approach is fruitful since it gives us insights into loops corrections which are difficult to handle on the string theory side.

2.3 The D-string/anti-D-string configuration

An argument corroborating this discover came from the analysis of loops corrections to the tachyon potential [20]. They found that the non-BPS D-brane configuration considered in [18] is not the minimum of the tachyon potential when the radii are critical. Instead, they argued that the true vacuum is obtained by giving a non vanishing vacuum expectation value for one of the four tachyon fields, say the state χ^9 (we still call them tachyon fields even if they are massless at critical radii). According to Sen [12], this corresponds to a splitting of the non-BPS D-brane into the D/$\bar{\mathrm{D}}$ pair with a relative $\mathbb{Z}_2$ Wilson line in which the non-BPS D-brane would decay when $R_9 > \sqrt{\alpha'/2}$. To summarize, the main result of [20] was to show that, for critical radii, while the analysis of the open string spectrum does not discriminate between the non-BPS D-brane and the D/$\bar{\mathrm{D}}$ phases, the minimization of the tachyon potential selects the latter state.

In the T-dual type IIB on $T^4/\mathcal{I}_4$ picture that we will use in the rest of this article, this vacuum is given by D-string/anti-D-string pairs sitting at opposite fixed points along the compact coordinate, all D-branes at one point ($\mathbf{x}_1$) and all anti-D-branes at the other ($\mathbf{x}_2$). Since we want to find the supergravity solution which describes this vacuum, we will first review the boundary state associated to this configuration. The (anti-)D-strings are fractional BPS (anti-)D-branes, which can be interpreted as (anti-)D3-branes wrapped on vanishing supersymmetric 2-cycles of the orbifold. The boundary state for the D-string reads

$$|D1\rangle = \frac{T_1}{2\sqrt{2V}}\left(|B1,\mathbf{x}_1\rangle^U_{\mathrm{NS-NS}} + |B1,\mathbf{x}_1\rangle^U_{\mathrm{R-R}}\right) + \frac{T_1}{2\sqrt{2}\pi^2\alpha'}\left(|T_1\rangle_{\mathrm{NS-NS}} + |T_1\rangle_{\mathrm{R-R}}\right) \tag{2.4}$$

while, for the anti-D-string, it is given by

$$|\bar{D}1\rangle = \frac{T_1}{2\sqrt{2V}}\left(|B1,\mathbf{x}_2\rangle^U_{\mathrm{NS-NS}} - |B1,\mathbf{x}_2\rangle^U_{\mathrm{R-R}}\right) - \frac{T_1}{2\sqrt{2}\pi^2\alpha'}\left(|T_2\rangle_{\mathrm{NS-NS}} - |T_2\rangle_{\mathrm{R-R}}\right), \tag{2.5}$$

where the explicit forms of the boundary states in the untwisted and twisted sectors can be found in [21].

These D-strings have opposite charges under the untwisted R-R 2-form. Since we will only discuss in this article the six-dimensional solutions, we will not see the dipole effect due to the separation of the branes along the orbifold directions. The boundary state we should consider to extract the couplings to the brane and the asymptotic behaviour of the supergravity fields is obtained by "neglecting" the dependence on the orbifold coordinates and summing these two boundary states; the untwisted R-R contributions cancel and it remains only:

$$|D1;\bar{D}1\rangle = \frac{T_1}{\sqrt{2V}}|D1\rangle^U_{\mathrm{NS-NS}} + \frac{T_1}{2\sqrt{2}\pi^2\alpha'}(|T_1\rangle_{\mathrm{NS-NS}} + |T_1\rangle_{\mathrm{R-R}} - |T_2\rangle_{\mathrm{NS-NS}} + + |T_2\rangle_{\mathrm{R-R}}). \tag{2.6}$$

Compared to the boundary state (2.3), the key difference is that the D-string/anti-D-string configuration also couples to the two twisted NS-NS scalars b^1 and b^2 localized at the fixed points $\mathbf{x}_1$ and $\mathbf{x}_2$. From the boundary states, we can extract the couplings of the D-branes to the closed string fields. However, we will refrain to give them now and postpone this presentation after the description of the six-dimensional supergravity action which governs the dynamics of the bulk fields.

3. Low energy effective actions

3.1 The six-dimensional supergravity action

To write the effective action which describes the dynamics of the fields excited by the branes, we will start from the ten dimensional action and perform a Kaluza-Klein reduction on the orbifold space $T^2/\mathbb{Z}_2$. This approach was also adopted in [21] for the IIA string theory, where a consistency check can be made by comparing with the action obtained through duality with heterotic string theory compactified on a 4-torus. Since type-IIA and type-IIB strings share the NS-NS untwisted and twisted sectors, we can read directly the corresponding kinetic terms in [21]. The difference lies in the R-R part and the Wess-Zumino term but these couplings can be easily extrapolated from the knowledge of their type IIA counterparts.

We start with the ten dimensional action of type-IIB string theory, which, in the string frame, reads:[2]

$$\mathcal{S}^{B,\sigma}_{10} = \frac{1}{2\kappa^2_{10}}\Bigg[\int d^{10}x\sqrt{-\det_{10} g}\, e^{-2\phi}R - \int e^{-2\phi}\left(4d\phi\wedge^\star d\phi + \frac{1}{2}H_{(3)}\wedge^\star H_{(3)}\right) - -\frac{1}{2}\left(dC_{(0)}\wedge^\star dC_{(0)} + \tilde{F}_{(3)}\wedge^\star\tilde{F}_{(3)} + \frac{1}{2}\tilde{F}_{(5)}\wedge^\star\tilde{F}_{(5)} + C_{(4)}\wedge H_{(3)}\wedge F_{(3)}\right)\Bigg], \tag{3.1}$$

[2]The gravitational coupling constant is given by $\sqrt{2}\kappa_{10} = (2\pi)^{7/2}\alpha'^2 g_s$. It includes the asymptotic value of the dilaton.

where the field strengths of the NS-NS 2-form and of the R-R 2- and 4-forms potential are defined as

$$H_{(3)} = dB_{(2)}\,, \qquad F_{(p+1)} = dC_{(p)} \qquad \text{and} \qquad \tilde{F}_{(p+1)} = F_{(p+1)} - C_{(p-2)} \wedge H_{(3)}\,.$$

The self-duality of $\tilde{F}_{(5)}$ is imposed at the level of the equations of motion.

As stressed in [18], when doing the Kaluza-Klein reduction, one should be careful with the background expectation values of the supergravity fields like the volume of the 4-torus since the stability of the D-brane system depends crucially on the value of this parameter. Therefore, we will separate the v.e.v. of the bulk fields from their fluctuations. We will not do the complete reduction since, from the boundary state analysis, we already know that only a subset of the fields will be active. Anticipating on the form of the solution, from the internal part of the metric, we keep only the fluctuations of the four scalars associated to its diagonal components out of the 58 parameters which parameterize the moduli space of metrics on K3:

$$g_{\mu\mu} = \mathrm{e}^{2\eta_\mu}\,, \qquad \mu = 6, \ldots, 9\,.$$

The angular deformations of the 4-torus and the moduli associated to the sixteen fixed points of the orbifold will not be considered. The untwisted sector can be obtained using the standard Kaluza-Klein reduction on a 4-torus and keeping only the states invariant under the orbifold projection. The only slightly subtle point is that the $\mathbb{Z}_2$ identification halves the volume of compactification. To write the twisted and Chern-Simmons contributions to the action, we use the expansions (2.1) of the NS-NS and R-R fields on the harmonic forms of the orbifold that we normalize in such a way that

$$\int_{\mathrm{K3}} \tilde{\omega}_2^I \wedge^{\star_4} \tilde{\omega}_2^J = (2\pi\sqrt{\alpha'})^4 \delta^{IJ}/2\,.$$

Including also the untwisted R-R 2-form $C_{(2)}$ (even if we know that the branes system is not charged under this field and, as we will see in the next section, it can be consistently put to zero), the reduction of (3.1) we need reads

$$\begin{aligned} \mathcal{S}_6^{B,\sigma} = {} & \frac{1}{2\kappa_{10}^2}\frac{V}{2}\Bigg[\int d^6x\sqrt{-\det_6 g}\,\mathrm{e}^{-2\varphi}R + \int 4\mathrm{e}^{-2\varphi} d\varphi \wedge^{\star_6} d\varphi - \\ & - \mathrm{e}^{-2\varphi} d\eta_a \wedge^{\star_6} d\eta_a - \frac{1}{2}F_{(3)} \wedge^{\star_6} F_{(3)}\Bigg] - \\ & - \frac{(2\pi\sqrt{\alpha'})^4}{4\kappa_{10}^2}\int \frac{1}{2}\Big[\mathrm{e}^{-2\phi}\mathcal{H}^I_{(1)} \wedge^{\star_6} \mathcal{H}^I_{(1)} + \tilde{\mathcal{G}}^I_{(3)} \wedge^{\star_6} \tilde{\mathcal{G}}^I_{(3)} + \mathcal{A}^I_{(2)} \wedge \mathcal{H}^I_{(1)} \wedge F_{(3)}\Big]\,. \end{aligned} \tag{3.2}$$

in the string frame. We have defined the fluctuations of the six-dimensional dilaton as

$$\varphi \equiv \phi - \frac{1}{4}\log(\det_4 g) = \phi - \frac{1}{2}\sum_{a=6}^{9}\eta_a$$

and the volume $V \equiv \prod_{a=6}^{9}(2\pi R_a)$ where the R_a are the radii of the 4-torus. In the following, we will forget the 6 in $\star_6$ since all bulk actions and equations will be six-dimensional. The Weyl rescaling

$$g_{\mu\nu} = e^{\varphi} G_{\mu\nu}\,,$$

leads us to the action in the six-dimensional Einstein frame:

$$\begin{aligned}\mathcal{S}_6^B = \frac{1}{2\kappa_{orb}^2}\Bigg[\int d^6x\sqrt{-\det G}R - \int\bigg(d\varphi\wedge^{\star}d\varphi + d\eta_a\wedge^{\star}d\eta_a + \frac{1}{2}\,\mathrm{e}^{\sum_a\eta_a}F_{(3)}\wedge^{\star}F_{(3)} + \\ + \mathrm{e}^{-\sum_a\eta_a}\mathcal{H}^I_{(1)}\wedge^{\star}\mathcal{H}^I_{(1)} + \frac{1}{2}\left(\tilde{\mathcal{G}}^I_{(3)}\wedge^{\star}\tilde{\mathcal{G}}^I_{(3)} + \sqrt{2}\,\mathcal{A}^I_{(2)}\wedge\mathcal{H}^I_{(1)}\wedge F_{(3)}\right)\bigg)\Bigg],\end{aligned} \tag{3.3}$$

where we have defined the six-dimensional gravitational coupling constant

$$\kappa_{orb}^2 = \frac{2\,\kappa_{10}^2}{V}$$

and we have rescaled the twisted fields[3] as

$$\mathcal{A'}^I_{(2)} = \frac{(2\pi\sqrt{\alpha'})^2}{\sqrt{V}}\,\mathcal{A}^I_{(2)}\,, \qquad b'^I = \frac{(2\pi\sqrt{\alpha'})^2}{\sqrt{2\,V}}\,b^I \tag{3.4}$$

in order to agree with the conventions of [21]. This step is not indispensable since an other convention would have manifested itself through different coupling constants in the boundary action and in the asymptotic values of the fields in the presence of the branes but it will allow us to blindly use the results of [21] that we will quickly review in the following part before tackling the core subject of this article.

3.2 Boundary actions and the asymptotic conditions

Using the boundary state which corresponds to a fractional Dp-brane and the methods developed in [28], the authors of [21] found the linear couplings of the bulk fields to a fractional D-brane and their asymptotic behaviour in the presence of such source terms. Since our configuration is a superposition of a fractional D-string and a fractional anti-D-string, we can easily get from their result the minimally covariantized linear couplings of the bulk fields to our D-strings pair. For a fractional Dp-brane which corresponds to a Dp+2-brane wrapped on the vanishing 2-cycle $\tilde{\omega}^I$ the effective action reads:

$$\begin{aligned}\mathcal{S}_{\mathrm{D}p} = -\frac{M_p}{2}\left\{\int d^{p+1}\sigma\sqrt{-\det\hat{G}}\;\mathrm{e}^{\frac{(p-1)\varphi}{2}-\sum_a\frac{\eta_a}{2}} - \int C_{(p+1)}\right\} - \\ -\frac{M_p\sqrt{V}}{4\pi^2\alpha'}\left\{\sqrt{2}\int d^{p+1}\sigma\sqrt{-\det\hat{G}}\mathrm{e}^{\frac{(p-1)\varphi}{2}-\sum_a\frac{\eta_a}{2}}b^I - \int\left[\mathcal{A}^I_{(p+1)} + \sqrt{2}b^I\,C_{(p+1)}\right]\right\}.\end{aligned} \tag{3.5}$$

[3] In equation (3.3), we have suppressed the ’ on the redefined fields to lighten the notations.

The coupling M_p is related to the standard D-brane tension:

$$M_p \equiv \frac{\sqrt{2}T_p}{\sqrt{V}\kappa_{\rm orb}}$$

and $\hat{G}$ is the two-dimensional induced metric. Looking at the boundary state (2.5), we see that we get the action for an anti-D-brane by changing the signs in front of the untwisted R-R form $C_{(p+1)}$ and the twisted NS-NS field b^I. Therefore, using the boundary state (2.6) and the action (3.5), it is easy to derive the action for the D/$\bar{\rm D}$-brane system:

$$\begin{aligned} \mathcal{S}_{\rm D1/\bar{D}1} = & -M_1 \int d^2\sigma \sqrt{-\det \hat{G}}\, {\rm e}^{-\sum_a \frac{\eta_a}{2}} + \frac{M_1\sqrt{V}}{4\pi^2\alpha'} \times \\ & \times \left[\int \left(\mathcal{A}^1_{(2)} + \mathcal{A}^2_{(2)} \right) - \sqrt{2} \int d^2\sigma \sqrt{-\det \hat{G}}\, {\rm e}^{-\sum_a \frac{\eta_a}{2}} \left(b^1 - b^2 \right) \right] . \end{aligned} \tag{3.6}$$

Doing similar calculations for the non-BPS D2-brane (2.3) stretched between the two fixed points $\mathbf{x}_1$, $\mathbf{x}_2$, leads to:

$$\mathcal{S}_{\rm non-BPS\ D2} = -\frac{M_1 R_9}{\sqrt{2\alpha'}} \int d^2\sigma \sqrt{-\det \hat{G}}\, {\rm e}^{\eta_9 - \sum_a \frac{\eta_a}{2}} + \frac{M_1\sqrt{V}}{4\pi^2\alpha'} \int \left(\mathcal{A}^1_{(2)} + \mathcal{A}^2_{(2)} \right) . \tag{3.7}$$

In this formula, $\hat{G}$ is also the two-dimensional induced metric on the non-compact longitudinal space of the D2-brane.

One can use at least two different methods to determine a supergravity solution. One is to solve the equations of motion obtained by varying the sum of the bulk and boundary actions. The other one is to determine the fields which solve the homogeneous equations which follow from (3.3) and have boundary conditions dictated by the presence of the brane. For the fractional Dp-brane, this asymptotic behaviour has also been calculated in [21]. Since we will adopt the second procedure in the next section, let us first recall their result and then, extend it to the non-BPS D-brane and to the D-string/anti-D-string configuration; at infinity, in the presence of a fractional Dp-brane, the metric, the scalars η_a, the dilaton and the untwisted R-R field have expansions which begin as

$$\begin{aligned} & G_{\alpha\beta} \sim \left(1 + \frac{(p-3)Q_p}{8r^{3-p}} \right) \eta_{\alpha\beta} , \quad G_{ij} \sim \left(1 + \frac{(p+1)Q_p}{8r^{3-p}} \right) \delta_{ij} , \\ & \eta_a \sim \frac{Q_p}{8r^{3-p}} , \qquad \varphi \sim \frac{(1-p)Q_p}{8r^{3-p}} , \qquad C_{(p+1)} \sim -\frac{Q_p}{2r^{3-p}} \end{aligned} \tag{3.8}$$

while the twisted fields behave as

$$b^I \sim -\frac{Q_p\sqrt{V}}{4\sqrt{2}\pi^2\alpha' r^{3-p}} , \qquad \mathcal{A}^I_{(p+1)} \sim -\frac{Q_p\sqrt{V}}{4\pi^2\alpha' r^{3-p}} , \tag{3.9}$$

where

$$Q_p = \frac{2M_p\kappa_{orb}^2}{(3-p)\Omega_{4-p}}. \tag{3.10}$$

From the boundary states (2.3) and (2.6) we can derive similar results for the non-BPS D2-brane and for the D-string/anti-D-string configuration. For the non-BPS D2-brane, one gets

$$\begin{aligned}
&G_{\alpha\beta} \sim \left(1 - \frac{Q_1}{2r^2}\frac{R_9}{\sqrt{2\alpha'}}\right)\eta_{\alpha\beta}\,, \qquad G_{ij} \sim \left(1 + \frac{Q_1}{2r^2}\frac{R_9}{\sqrt{2\alpha'}}\right)\delta_{ij}\,,\\
&\eta_a \sim \frac{Q_1}{4r^2}\frac{R_9}{\sqrt{2\alpha'}}\,, \qquad \text{for } a = 6,\ldots,8, \qquad \eta_9 \sim -\frac{Q_1}{4r^2}\frac{R_9}{\sqrt{2\alpha'}}\\
&\mathcal{A}^I_{(2)} \sim -\frac{Q_1\sqrt{V}}{4\pi^2\alpha' r^2}\,, \qquad \text{for } I = 1,2\,,
\end{aligned} \tag{3.11}$$

and, for the D-string/$\bar{\text{D}}$-string,

$$\begin{aligned}
&G_{\alpha\beta} \sim \left(1 - \frac{Q_1}{2r^2}\right)\eta_{\alpha\beta}\,, \qquad G_{ij} \sim \left(1 + \frac{Q_1}{2r^2}\right)\delta_{ij}\,, \qquad \eta_a \sim \frac{Q_1}{4r^2}\,,\\
&\mathcal{A}^I_{(2)} \sim -\frac{Q_1\sqrt{V}}{4\pi^2\alpha' r^2}\,, \qquad b^I \sim -(-1)^I\frac{Q_1\sqrt{V}}{4\sqrt{2}\pi^2\alpha' r^2}\,, \qquad \text{for } I = 1,2\,.
\end{aligned} \tag{3.12}$$

In the next section, we will use these results to discuss the no-force condition at leading order in Q_1 and solve the equations of motion with the boundary conditions (3.12).

4. Supergravity description of the D-string/anti-D-string configuration

4.1 No-force condition and critical radii

One key ingredient in the construction of supergravity solutions which describe BPS Dp-branes is that these branes do not exercise any force on each other. This property, which follows from the presence of supersymmetry, allows to construct a bound state of an arbitrary number of D-branes and then to obtain a solution with small curvature. Therefore, α' curvature corrections to the supergravity action can be safely neglected. This no-force condition is a desirable feature for any type of branes, including the non-BPS ones, in order to find a reliable classical solution of the low energy effective action of a string theory. In the supergravity picture, such a constraint can be tested by calculating the potential a non-BPS brane probe feels in the geometry created by a bound state of similar branes.

We can already investigate this no-force constraint for the asymptotic part of the fields we have derived in the last section. To do this, we insert these fields into

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the action of a brane probe and evaluate the potential. For the non-BPS D2-brane, one gets:

$$\mathcal{S}_{\rm probe} = -M_1 \int d^2\sigma \left[\frac{R_9}{\sqrt{2\alpha'}} - \frac{R_9^2}{2\alpha'}\frac{Q_1}{r^2} + \frac{2V}{(4\pi^2\alpha')^2}\frac{Q_1}{r^2} \right] . \tag{4.1}$$

Therefore, the potential is constant when $\alpha' R_9 = 4R_6R_7R_8$. When all the radii are critical, this relation is verified and we recover the one-loop no-force condition. However, we also see that, when $\alpha' R_9 < 4R_6R_7R_8$, which is always the case for a *stable* non-BPS D2-brane configuration with at least one non critical radius, the force is repulsive. Therefore, it is impossible to bring at no cost these non-BPS D2-branes from infinity in order to form a macroscopic bound state whose supergravity description will be reliable. Finally, if we want to discuss only configurations which are stable in the string theory description, the only possibility is for critical radii. Its T-dual counterpart has been analyzed in [18], where they found that the no-force condition is broken by higher loops string corrections and the force is always repulsive.

The same analysis for the D-string/$\bar{\rm D}$-string configuration gives a potential

$$\begin{aligned}\mathcal{S}_{\rm probe} &= -M_1 \int d^2\sigma \left[1 - \frac{Q_1}{r^2} + \frac{2V}{(4\pi^2\alpha')^2}\frac{Q_1}{r^2} - \frac{2V}{(4\pi^2\alpha')^2}\frac{Q_1}{r^2} \right] \\ &= -M_1 \int d^2\sigma \left[1 - \frac{Q_1}{r^2} \right]\end{aligned} \tag{4.2}$$

which does not depend on the moduli of the four-torus. Actually, one sees that, in each twisted sector, the NS-NS and R-R contributions cancel each other and only remains the attractive potential associated to the untwisted NS-NS sector. This result is consistent with the analysis of [20] where they found that two D/$\bar{\rm D}$-branes systems always attract each other. Despite being better than the repulsive force found in [18] which forbids them to construct a macroscopic bound state of non-BPS branes, one may worry about the absence of a no-force condition. However, so far, we have only considered the long distance leading behaviour of the fields in the presence of the D/$\bar{\rm D}$ pair and it is possible that at shorter distances other phenomenons occur. For instance, one can imagine that the force vanishes at a finite distance. But, to investigate this question, it is necessary to go beyond the leading linear behaviour we have discussed so far and to find the complete solution.

4.2 The solution

We will now solve the equations of motion which follow from the action (3.3) with the boundary conditions (3.12). To get a reliable solution at weak string coupling, we should assume that a macroscopic bound state made of N D/$\bar{\rm D}$ pairs can be constructed. Since these D-branes interact among them in a non trivial way, it is likely that the couplings of the bound state to the supergravity fields, specially to the NS-NS sector, are not just given by the sum of N D/$\bar{\rm D}$ sources (3.6). Actually, in the appendix, we provide the solution for arbitrary boundary conditions of the bulk fields, the only hypothesis being that the bound state excites the same supergravity fields as its components. However, for simplicity and since we don't know the actual couplings of our interacting system, we will limit our discussion in this section to the debatable hypothesis that they are given by the naive sum of N boundary actions (3.6) and we will comment on this assumption at the end.

First, since we do not have any source term for the dilaton and for the untwisted R-R field $C_{(2)}$ and since we saw that the leading contributions to their asymptotic behaviour cancel, we choose to put them to zero. The dilaton equation, which is simply

$$d^\star d\varphi = 0\,, \tag{4.3}$$

will be trivially verified. Moreover, it is also easy to see that the equation for the untwisted R-R field is satisfied for an ansatz which preserves the symmetry of the problem, namely identical twisted R-R potentials and opposite twisted NS-NS scalars. The other equations of motion read:

$$d^\star d\eta_a + \frac{1}{2}\sum_{I=1,2} \mathrm{e}^{-\sum_a \eta_a} \mathcal{H}^I_{(1)} \wedge {}^\star\mathcal{H}^I_{(1)} = 0 \tag{4.4}$$

for the moduli η_a $(a = 1, \ldots, 4)$,

$$d^\star(\mathrm{e}^{-\sum_a \eta_a} \mathcal{H}^I_{(1)}) = 0\,, \qquad I = 1, 2 \tag{4.5}$$

for the NS-NS twisted scalars b^I,

$$d^\star \tilde{\mathcal{G}}^I_{(3)} = 0\,, \qquad I = 1, 2 \tag{4.6}$$

for the R-R twisted $\mathcal{A}^I_{(2)}$ and

$$R_{\mu\nu} - \partial_\mu\eta_a\partial_\nu\eta_a - \mathrm{e}^{-\sum_a \eta_a} \partial_\mu b^I \partial_\nu b^I - \frac{1}{4}\, \tilde{\mathcal{G}}^I_{(3)\,\rho\sigma\mu} \tilde{\mathcal{G}}^{I\,\rho\sigma}_{(3)\,\nu} = 0 \tag{4.7}$$

for the metric. We introduce the following ansatz which depends only on the radial coordinate and is compatible with the symmetries of the source:

$$\begin{aligned}
&ds^2 = B^2(r)(-(dx^0)^2 + (dx^1)^2) + F^2(r)(dr^2 + r^2 d\Omega_3^2)\,, \quad r^2 = (x^2)^2 + \ldots + (x^5)^2\,,\\
&\tilde{\mathcal{G}}^I_{(3)} = dA(r) \wedge dx^0 \wedge dx^1 - {}^\star(dA(r) \wedge dx^0 \wedge dx^1)\,,\\
&b^I(r) = (-1)^I b(r) \qquad \text{and} \qquad \eta_a(r) = \eta(r)\,.
\end{aligned} \tag{4.8}$$

After some calculations which are detailed in the appendix A, one arrives at the solution. The fields can be separated into two groups, reflecting the splitting of the equations into two almost independent sets. The scalars η and the NS-NS twisted fields b read:

$$\eta = -\frac{1}{2}\ln\left[\frac{\cosh\Phi}{\sqrt{1+\alpha^2}}\right], \qquad b = -\frac{1}{\sqrt{2}}\left[\alpha + \sqrt{1+\alpha^2}\tanh\Phi\right], \tag{4.9}$$

where we have defined the parameter $\alpha \equiv \sqrt{V_c/2V}$ which depends on the critical volume $V_c \equiv \prod_a (2\pi R_a^c) = 2(2\pi^2\alpha')^2$ and the functions of the radial coordinates:

$$\Phi \equiv \frac{\sqrt{1+\alpha^2}\, Qy}{2\alpha} - \operatorname{arc\,sinh}\alpha \qquad \text{and} \qquad y(r) \equiv \frac{2\sqrt{3}}{Q} \operatorname{arctanh}\left(\frac{Q}{2\sqrt{3}\, r^2}\right). \quad (4.10)$$

The form of the metric and of the twisted R-R fields depends on the parameter α. For a volume strictly larger than $V_c/2$, we get:

$$A = \sqrt{1-\alpha^2}\,\operatorname{cotan}\Theta - \alpha\,, \qquad B^2 = \frac{\sqrt{1-\alpha^2}}{\sin\Theta}\,, \qquad F^2 = f_+ f_- B^{-2}\,, \qquad (4.11)$$

where we have introduced:

$$\Theta \equiv \frac{\sqrt{1-\alpha^2}\, Qy}{2\alpha} + \arccos\alpha \qquad \text{and} \qquad f_\pm(r) \equiv 1 \pm \frac{Q}{2\sqrt{3}\, r^2}\,. \qquad (4.12)$$

For a volume smaller than $V_c/2$, one must replace in (4.11) the trigonometric functions by their hyperbolic counterparts and change $\sqrt{1-\alpha^2}$ to $\sqrt{\alpha^2-1}$:

$$\begin{aligned} &A = \sqrt{\alpha^2-1}\,\operatorname{cotanh}\tilde{\Theta} - \alpha\,, \qquad B^2 = \frac{\sqrt{\alpha^2-1}}{\sinh\tilde{\Theta}}\,, \qquad F^2 = f_+ f_- B^{-2}\,, \\ &\tilde{\Theta} \equiv \frac{\sqrt{\alpha^2-1}\, Qy}{2\alpha} + \operatorname{arccosh}\alpha\,. \end{aligned} \qquad (4.13)$$

Finally, the solution at $V_c/2$ can be obtained as a degenerate limit of these two cases:

$$A = -\frac{Qy}{2+Qy}\,, \qquad B^2 = \frac{2}{2+Qy}\,, \qquad F^2 = f_+ f_- B^{-2}\,. \qquad (4.14)$$

Let us discuss the properties of these solutions. First, we can notice that the functions $f_\pm$ and y do not depend on the volume: the reason lies in a cancellation between the contributions of the twisted R-R and NS-NS sectors. The second observation we can make is that the "transition" between solutions (4.11) and (4.13) does not occur at the critical volume, contrary to one can have naively expected. However, we have to remind the reader that this result is related to our working hypothesis, namely that the couplings of the bound state are given by the naive sum of its components. The third point we would like to emphasize is that the solution depends on the radii only through the volume of the four-torus. This means that it can not capture the detailed structure of the stability domain of the non-BPS system. For instance, when $V > V_c$, the same supergravity solution will describe configurations which are stable and configurations which are unstable from the string theory point-of-view. Therefore, we don't expect to see transitions between stable and unstable configurations in these solutions. This remark can also explain why the critical volume does not seem to play any special role.

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Then, one can calculate the scalar curvature

$$R = \left(1 + \frac{1}{\alpha^2}\right) \frac{B^2}{(r^2 f_+ f_-)^3} \tag{4.15}$$

and investigate the presence of possible singularities.

In the case $V > V_c/2$, the metric and the twisted R-R fields have branch cut singularities at radii defined by

$$y_n \equiv y(r_n) = \frac{2\alpha(n\pi - \arccos\alpha)}{Q\sqrt{1-\alpha^2}}, \qquad n = 1, 2, \ldots$$

where the longitudinal components of the metric and the curvature (4.15) diverge. Moreover, the first of these singularities deserves the name of a "repulson" since the newtonian gravitational force becomes repulsive in the region $]r_1, r_r[$ where the radius r_r defined by

$$y(r_r) = \frac{2\alpha(\pi/2 - \arccos\alpha)}{Q\sqrt{1-\alpha^2}}\,.$$

When the volume becomes equal to or larger than $V_c/2$, these branch cut singularities disappear and only remains a singularity located at $y(r_0) = \infty$ or, using (4.10), at a radius

$$r_0^2 = Q/2\sqrt{3}\,.$$

This singularity is rather different from the other singularities obtained for $V > V_c/2$ since now, the longitudinal components of the metric vanish as one reaches it. The curvature divergence is due to the presence of the function f_- in the scalar curvature. Finally, this singularity is always attractive.

These naked singularities make the solutions unacceptable according to the cosmic censorship. However, we know D-brane configurations which also give rise to singularities but, in some cases, this pathological behaviour can be solved. An example is the enhançon mechanism [22] which solves the singularity of the metric which corresponds to fractional D-branes. In the next subsection, we will argue for a similar resolution of the singular solutions generated by D/$\bar{\text{D}}$ pairs.

4.3 D-brane probe and the enhançon mechanism

To fix the ideas, we will consider only the case $\alpha > 1$ but the other two cases are completely similar as we will comment at the end. To understand what is happening when, falling from infinity, we are approaching the first of these singularities, we consider a probe D-string/$\bar{\text{D}}$-string pair slowly moving in the geometry defined by (4.9) and (4.11). In the static gauge, its space-time coordinates are $\zeta^\alpha = x^\alpha$ and $\zeta^i = \zeta^i(x^0)$. Its velocity is given by $v_i = \dot{\zeta}^i$. Then, using (4.9) and (4.11) in the boundary action (3.6), and expanding in powers of v^2, we get

$$\mathcal{S}_{\text{probe}} = -M_1 \int d^2\sigma \left[1 - \frac{\sqrt{1-\alpha^2}\,(\cos\Theta + \sinh\Phi)}{\alpha\sin\Theta} + \frac{f_+ f_- \sin\Theta\ \sinh\Phi\ v^2}{2\alpha\sqrt{1-\alpha^2}} + \mathcal{O}(v^4)\right]. \tag{4.16}$$

The v independent term corresponds to the potential felt by the probe in the supergravity background. Expanding this term when $r \to \infty$, one obviously recovers the attractive force we had already observed at infinity (4.2). One can also verify that this force remains attractive in the whole region $]r_1, +\infty[$.[4]

We also observe that the sign of the v^2 dependent part changes at a radius $r_e > r_1$ defined by $\Phi = 0$:

$$y(r_e) = \frac{2\alpha \operatorname{arcsinh} \alpha}{Q\sqrt{1+\alpha^2}} \,. \tag{4.17}$$

The D/$\bar{\mathrm{D}}$-string pair falling from infinity becomes tensionless before reaching the singularity. The vanishing of the effective tension can be traced back to the non trivial contributions of the twisted NS-NS fields b^1 and b^2 to the Dirac-Born-Infeld part of the action. Indeed, at infinity, the fluctuations of these fields vanish (we remind the reader that the *vev* of the twisted NS-NS fields are 1/2 in this compactification). However, when we approaching the core, these fluctuations decrease and exactly compensate their background value at r_e.

The existence of a tensionless sphere is reminiscent of the situation studied in [22] and we can suggest a similar resolution of the singularity; the radius r_e at which the probe becomes tensionless was called the enhançon radius in [22]: the reason was that, at this radius, new massless bulk states appeared and the gauge group symmetry was enhanced.

To see if a similar enhançon mechanism can also be argued for solving the singularity of our solution, it is convenient to discuss the theory from a type-IIA on $T^4/\mathbb{Z}_2$ point-of-view. Therefore, we T-dualize our solution along a direction transverse to $K3$ and to the D-strings. The dual type-IIA string theory on $T^4/\mathbb{Z}_2$ has a gauge group $\mathrm{U}(1)^{24}$, with 16 U(1) associated to the R-R twisted 1-forms $\mathcal{A}^I_{(1)}$, which come from the reduction of the R-R 3-form $C_{(3)}$ on the vanishing supersymmetric 2-cycles $\tilde{\omega}^I_2$, 6 U(1) associated to the R-R untwisted 1-forms $C^i_{(1)}$, obtained by reducing $C_{(3)}$ on the 2-cycles of the torus and 2 U(1) whose gauge potentials are the R-R 1-form $C_{(1)}$ and the six-dimensional Hodge-dual of $C_{(3)}$. After T-duality, the D-string/$\bar{\mathrm{D}}$-string system becomes a D2-brane/$\bar{\mathrm{D}}$2-brane configuration, which is magnetically charged under two twisted R-R 1-forms $\mathcal{A}^1_{(1)}$ and $\mathcal{A}^2_{(1)}$. Therefore, our D2-brane/$\bar{\mathrm{D}}$2-brane system appears as a non-BPS monopole of the gauge group $\mathrm{U}(1)_1 \otimes \mathrm{U}(1)_2 \subset \mathrm{U}(1)^{24}$. The twisted NS-NS fields remain the same.

At the enhançon radius, one can notice that an enhancement of these gauge groups occurs. This enhancement is due to fractional D0-branes, electrically charged under $\mathcal{A}^1_{(1)}$ or $\mathcal{A}^2_{(1)}$, which become massless at r_e. To see this, consider the action for such fractional D0-branes, i.e. eqn. (3.5) with $p = 0$; expanding the Dirac-Born-Infeld

[4] Since the force is attractive, the velocity and the energy of the probe can not be kept constant at the same time. Therefore, one may worry about the use of a non-relativistic approximation for the probe action. Actually, one can repeat the above analysis for a relativistic probe, like in [29], and verify that the conclusion remains unchanged.

part of the action to lowest order in the velocity, we obtain the following kinetic term:

$$\mathcal{S}_{\text{kin.}} = -\frac{M_0}{2} \int d\sigma\, \frac{f_+ f_- \sin\Theta \sinh\Phi}{2\alpha\sqrt{1-\alpha^2}}\, v^2 \tag{4.18}$$

which is obviously proportional to the one of the D-string/$\bar{\text{D}}$-string probe (4.16). Again, the vanishing of the effective tension is completely determined by the fluctuations of the twisted NS-NS fields, b^1 and b^2, and occurs when these fluctuations compensate their background values. Therefore, the masses of the (anti-)fractional D-particles located at the fixed points $\mathbf{x}_I, I = 1, 2$ vanish at r_e and, similarly to [22], we can conclude that, at r_e, stringy phenomena such as the enhancement of the gauge symmetry play a role and that the solution inside this special radius should not be taken seriously. Our supergravity solution should only be taken as physical down to the sphere of radius r_e and the N D/$\bar{\text{D}}$-string pairs live on this sphere.

Using the S-duality which relates type-IIA string theory compactified on $T^4/\mathbb{Z}_2$ and heterotic string on T^4, we can also give an heterotic interpretation of this gauge symmetry enhancement. The precise mapping of the moduli and of the gauge fields given in [30] tells us that the 16 twisted NS-NS moduli b^I are identified to Wilson lines values on one of the four compact directions of the 16 $U(1)$ fields, A^K, which form the Cartan torus of the broken $SO(32)$ group on the heterotic side. Using the mass formula for heterotic states given for example in [15],

$$\begin{aligned} \frac{1}{4} m_h^2 &= P_L^2 + 2(N_L - 1)\,, \\ P_L &= \left(V^K + A_a^K w^a, \frac{p_a}{2R_a} + w^a R_a \right), \quad p_a = n_a + B_{ab} w^b - V^K A_a^K - A_a^K A_b^K \frac{w^b}{2}\,, \end{aligned}$$

it is easy to verify that, when one of these values vanishes, new massless states appear and the associated U(1) gauge group is enhanced to SU(2). For instance, when the first two twisted NS-NS moduli are shifted by $-1/2$, these additional massless gauge bosons are given by $N_L = 0$, $w^a = 0, p_a = 0$ for all a and charges $V = \pm(1, \pm 1, 0^{14})$. We remind the reader that the sixteen twisted U(1) charges in the type-IIA picture correspond to symmetric and antisymmetric combinations of the $(2n + 1)$'st and $(2n + 2)$'nd U(1) charges in the heterotic description.

Since this mechanism only depends on the twisted NS-NS fields, whose form is independent of the parameter α, it will also work for $V \leq V_c/2$. The resolution of the singularities for any volume may seem strange since the string theory analysis predicts a tachyonic instability when $V < V_c$. However, as we have already argued, our supergravity description can not capture the transitions between stable and instable configurations, as it is shown by the cases $V > V_c$ which cover both situations. We have also to keep in mind the two hypothesis we have made to find this solution: first, we have neglected the dipole effects due to the separation of the D-branes along the compact directions. Taking into account this effect would probably lead to a solution which depends on the different radii not just through the volume. Therefore, it would be possible to investigate its behaviour under variations of the radii.

The second hypothesis was to take the sum of N D/$\bar{\text{D}}$ pair actions as a source. It is instructing to relax this hypothesis and see if something special happens for specific values of the couplings of the bound state to the bulk fields. The total charges of the system under the R-R twisted fields are always given by the sum of N charges and their asymptotic values are governed by $A_\infty = Q\sqrt{V/2V_c}$ but the couplings to the NS-NS fields can be modified. Let us consider generic asymptotic values for the metric, η_a and b^I (respectively called a, η_∞ and b_∞ in the appendix.). First, the nature of the singularities is governed by $\gamma \equiv a^2 - A_\infty^2$. When $\gamma < 0$, we have always branch cut singularities while when $\gamma \geq 0$, the parameter $c = \sqrt{(\gamma + 4\eta_\infty^2 + 2b_\infty^2)/6}$ introduced in the appendix is always a non zero real number and there is only one singularity, located at $y(r) = \infty$. The radius of the enhançon sphere is governed by the parameters a and b_∞. At fixed a, its position, y_e, decreases from ∞ to 0 (the corresponding radius increases) when b_∞ goes from 0 to ∞. In particular, the case $b_\infty = 0$ reproduces the T-dual version of the result of [18]. Therefore, for $\gamma \geq 0$, the singularity is always inside the enhançon sphere and coincides with it at the limit $b_\infty \to 0$.

On the other hand, when $\gamma < 0$, for given values of a and η_∞, at small enough b_∞, the first singularity will be outside the enhançon sphere. In this case, its naked singularities can not be resolved by the enhançon mechanism. Moreover, one can see that it is impossible to create a horizon just by playing on the values of the three parameters while staying in this situation. The non-BPS D-brane phase of [18] always corresponds to this latter case.

5. Discussion

In this paper, we have found the supergravity description of D-strings/$\bar{\text{D}}$-strings located at opposite fixed points of the orbifold IIB/$(T^4/\mathbb{Z}_2)$. These D-pairs feel an attractive force, while the force was repulsive for the non-BPS D-branes. The other important difference with the non-BPS D-brane is that the D/$\bar{\text{D}}$ pair is a source for twisted NS-NS scalars. Our solutions display singularities but we have observed that a probe in this background becomes tensionless before reaching the first singularity, suggesting a enhançon mechanism similar to the one advocated in [22]. Actually, we have shown that, on the tensionless sphere, new massless gauge fields appear and that the gauge group is enhanced. However, some questions are still open; indeed, it is impossible to see the effect of the tachyonic instability predicted by string theory in our solutions. This problem can be related to our working hypothesis. Indeed, the transitions between different D-branes phases depend on the value of each compactification radius, something that our solution does not capture.

Finally, let us speculate on how supergravity would select the "correct" solution. First, we expect that it will have no pathology and therefore that its singularities have been resolved. Then, the well-behaved solutions differ by their couplings to the NS-NS fields, which, due to interactions between the D-branes, can not be fixed easily. However, one could expect that the ground state would correspond to the solution with the smallest ADM mass.

Such solutions can have applications in non-supersymmetric versions of the AdS/CFT correspondence [31, 32]. In this context of dual description of D-brane/anti-D-brane pairs, let us conclude with some comments on a related but probably harder question which has been raised by Mukhi and Suryanarayana [33]. In the type-IIA picture, they have considered adjacent D4-branes/$\bar{\text{D}}$4-branes pairs stretched between two relatively rotated NS5-branes. They said that they expected a repulsive force between the endpoints of the adjacent brane and anti-brane but, since the D-branes can not be separated without being stretched, they argued that it should exist a critical value of the distance between the branes endpoints such that the repulsive force is compensated by the tension. In the T-dual type-IIB language, this system is interpreted as fractional D3-branes/$\bar{\text{D}}$3-branes pairs sitting at the fixed point of a conifold. The fractional D3-branes are wrapped D5-branes on one of the two 2-spheres of the base of the conifold, $T^{1,1}$. Separating the branes in the type-IIA picture may correspond to a deformation of the conifold; in particular, for symmetric reasons, the wrapped D5-brane and anti-D5-brane should be located at antipodal points of the other 2-sphere and the cancellation argued in [33] should demonstrate itself as the existence a radius of the sphere for which the system is stable. However, it is likely that the repulsive force gets higher loops corrections which are difficult to calculate in string theory. One can think that, as in [18], the classical description will tell us something about these corrections but this question is certainly difficult to solve since it involves dipole solutions which are not well understood so far.

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A. Resolution of the field equations

In this appendix, we solve explicitly for the ansatz (4.8) the equations of motion complemented by the generic boundary conditions:

$$B^2 \sim 1 - \frac{a}{r^2}\,, \quad G^2 \sim 1 + \frac{a}{r^2}\,, \quad \eta \sim \frac{\eta_\infty}{r^2}\,, \quad A \sim -\frac{A_\infty}{r^2}\,, \quad \text{and} \quad b \sim -\frac{b_\infty}{r^2}\,. \quad (A.1)$$

Identifying the asymptotic constants as

$$a = \frac{Q}{2}\,, \quad \eta_\infty = \frac{Q}{4}\,, \quad b_\infty = \frac{Q}{2}\sqrt{\frac{V}{V_c}} \quad \text{and} \quad A_\infty = \frac{Q}{\sqrt{2}}\sqrt{\frac{V}{V_c}}\,. \tag{A.2}$$

we will recover the boundary conditions (3.12). Introducing the field $\xi \equiv 2(\ln B + \ln F)$, the field equations become

$$\partial_r \left(r^3 \mathrm{e}^{\xi - 4\eta} \partial_r b^I \right) = 0 \tag{A.3}$$

for the NS-NS twisted scalars,

$$\partial_r \left(r^3 \mathrm{e}^{\xi} B^{-4} \partial_r A^I \right) = 0 \tag{A.4}$$

for the R-R twisted 2-forms,

$$\partial_r \left(r^3 \mathrm{e}^{\xi} \partial_r \eta \right) + r^3 \mathrm{e}^{\xi - 4\eta} \left(\partial_r b \right)^2 = 0 \tag{A.5}$$

for the NS-NS untwisted scalars and

$$\begin{aligned} R^r{}_r - \frac{1}{F^2}\left(4(\partial_r \eta)^2 + 2\mathrm{e}^{-4\eta}(\partial_r b)^2 - \frac{1}{2B^4}(\partial_r A)^2 \right) &= 0\,, \\ R^\theta{}_\theta - \frac{1}{2F^2 B^4}(\partial_r A)^2 &= 0\,, \\ R^\alpha{}_\alpha + \frac{1}{2F^2 B^4}(\partial_r A)^2 &= 0\,, \end{aligned} \tag{A.6}$$

for the radial, transverse and longitudinal components of the Ricci tensor. The first two equations, which govern the dynamics of the twisted fields, can be integrated to:

$$\begin{aligned} \partial_r b &= \frac{2b_\infty}{r^3}\, \mathrm{e}^{-\xi + 4\eta}\,, \\ \partial_r A &= \frac{2A_\infty}{r^3} B^4 \mathrm{e}^{-\xi}\,, \end{aligned} \tag{A.7}$$

where the boundary constraints (A.1) have been used to determine the constants of integration. Then, one reinjects these values in the formula (A.5) and (A.6). Equation (A.5) becomes:

$$\partial_r \left(r^3 \mathrm{e}^{\xi} \partial_r \eta \right) + \frac{4b_\infty^2}{r^3}\, \mathrm{e}^{-\xi + 4\eta} = 0 \tag{A.8}$$

that can be integrated to

$$\frac{r^3}{2}(\partial_r \eta)^2 + \frac{b_\infty^2}{r^3} \mathrm{e}^{-2\xi + 4\eta} = \frac{b_\infty^2 + 2\eta_\infty^2}{r^3}\, \mathrm{e}^{-2\xi}\,. \tag{A.9}$$

Using equations (A.7) and (A.9) in the Einstein equations (A.6), we get

$$\begin{aligned}
\Big[-\partial_r(r^3\partial_r \ln F) + r^3\partial_r\xi\partial_r \ln F - r^3\partial_r^2\xi -& \\
-2r^3\left((\partial_r \ln B)^2 + (\partial_r \ln F)^2\right)\Big] -& \\
-\frac{8(b_\infty^2 + 2\eta_\infty^2)}{r^3}\mathrm{e}^{-2\xi} + \frac{2A_\infty^2}{r^3}B^4\mathrm{e}^{-2\xi} &= 0\,, \qquad \text{(A.10)}\\
\left[-\partial_r(r^3\mathrm{e}^\xi\partial_r \ln F) - r^2\partial_r\mathrm{e}^\xi\right] - \frac{2A_\infty^2}{r^3}B^4\mathrm{e}^{-\xi} &= 0\,, \qquad \text{(A.11)}\\
\left[-\partial_r(r^3\mathrm{e}^\xi\partial_r \ln B)\right] + \frac{2A_\infty^2}{r^3}B^4\mathrm{e}^{-\xi} &= 0\,, \qquad \text{(A.12)}
\end{aligned}$$

where we have also introduced the explicit form of the Ricci tensor. We see that the equations split into two almost independent groups which are only connected by the function ξ: one for the six-dimensional metric and the twisted NS-NS fields, and the other for the untwisted NS-NS scalars and the twisted R-R forms.

The strategy is to first determine the field ξ and then solve the other equations separately. Summing the last two equations, we see that ξ obeys the homogeneous differential equation

$$\partial_r(r^5\partial_r\mathrm{e}^\xi) = 0 \qquad \text{(A.13)}$$

whose solution is of the form

$$\mathrm{e}^\xi = f_+(r)f_-(r) \qquad \text{(A.14)}$$

with

$$f_\pm(r) = 1 \pm \frac{c}{r^2}\,. \qquad \text{(A.15)}$$

The constant of integration c will be determined at the end of the calculation, by inserting into equation (A.10) the solutions we will find below. It is easy to see that the fields B and η can be expressed in term of the function:

$$y(r) \equiv \frac{1}{2c}\ln\left(\frac{f_+(r)}{f_-(r)}\right)\,. \qquad \text{(A.16)}$$

In term of this new variable, equations (A.8) and (A.12) simply read

$$\partial_y^2\eta + b_\infty^2\mathrm{e}^{4\eta} = 0 \qquad \text{(A.17)}$$

and

$$2\partial_y^2 \ln B - A_\infty^2 B^4 = 0 \qquad \text{(A.18)}$$

which are respectively solved by

$$\eta = -\frac{1}{2}\ln\left(\frac{\cosh\left(\sqrt{2}b_\infty\sqrt{1+\alpha^2}\,y - \operatorname{arcsinh}\alpha\right)}{\sqrt{1+\alpha^2}}\right) \qquad \text{(A.19)}$$

and

$$B^2 = \frac{\sqrt{\beta^2 - 1}}{\sinh\left(A_\infty \sqrt{\beta^2 - 1}\, y + \operatorname{arccosh} \beta\right)} \tag{A.20}$$

where the appropriate boundary conditions (A.1) have been used and we have defined

$$\alpha \equiv \frac{\sqrt{2}\eta_\infty}{b_\infty} \qquad \text{and} \qquad \beta \equiv \frac{a}{A_\infty}\,. \tag{A.21}$$

Then, to obtain the twisted fields, we integrate (A.7). The result is

$$\begin{aligned} b &= -\frac{\alpha + \sqrt{1+\alpha^2}\tanh\left(\sqrt{2}b_\infty\sqrt{1+\alpha^2}\, y - \operatorname{arcsinh}\alpha\right)}{\sqrt{2}}\,, \\ A &= -\beta + \sqrt{\beta^2 - 1}\,\operatorname{cotanh}\left(A_\infty\sqrt{\beta^2-1}\, y + \operatorname{arccosh}\beta\right). \end{aligned} \tag{A.22}$$

Finally, inserting these formula into equation (A.10) allows us to determine the constant

$$c = \sqrt{\frac{1}{6}\left(a^2 + 2b_\infty^2 + 4\eta_\infty^2 - A_\infty^2\right)}. \tag{A.23}$$

In section 4, we specialize this general result to the boundary conditions (A.2).

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